EDUCATIONAL MONOGRAPH

Prepared From

<u>Formulas for Radiant Heat Transfer Between Nongray Parallel Plates of Polished Refractory Metals</u>

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January, 1967

NASA Office of Technology Utilization
Contract NSR-37-002-045

FOREWORD

This Monograph was produced in a pilot program at Oklahoma State University in Stillwater, Oklahoma, under contract to the NASA Office of Technology Utilization. The program was organized to determine the feasibility of presenting the results of recent research in NASA Laboratories and under NASA contract in an education format suitable as supplementary material in classwork at engineering colleges. The Monograph may result from editing single technical reports or synthesizing several technical reports resulting from NASA's research efforts.

Following the preparation of the Monographs, the program includes their evaluation as educational material in a number of universities throughout the country. The results of these individual evaluations in the classroom situation will be used to help determine if this procedure is a satisfactory way of speeding research results into engineering education.

INSTRUCTOR'S GUIDE FOR MONOGRAPHS

1. Educational level of the Monograph--Undergraduate student in general heat transfer course.

2. Prerequisite course material--This Monograph is assumed to be used as a course addition after the presentation of the typical radiant heat exchange section of the undergraduate course.

3. Estimated number of lecture periods required--One half to one lecture period.

4. Technical significance of the Monograph--This Monograph introduces some of the practical techniques under consideration for nongray radiant exchange analysis. The attempts to reduce computational complexity indicate the desires of the practical analyst.

5. New concepts or unusual concepts illustrated--This Monograph introduces a nongray technique in practical use.

6. How Monograph can best be used--
   (a) A short lecture should be given in class
   (b) Class problems should be assigned to ensure students attention to the reading material.

7. Other literature--Chapter 4 of Heat Transfer by Max Jakob; Chapter 6 of Engineering Radiation Heat Transfer by J. A. Wiebelt.

8. Who to contact for further information--Technical Utilization Officer, Lewis Research Center, Cleveland, Ohio.

9. Note to instructor: All uncolored pages of the instructors Monograph are in the copies intended for student use.

ABSTRACT

Hemispherical emittance, both total and spectral were calculated from normal spectral-emittance data. The metals evaluated were clean polished tungsten, molybdenum, and tantalum, each of which exhibits spectral emittances that vary considerably with temperature and wavelength.

Net radiant heat flow between two parallel infinite plates was computed by summing the monochromatic energy exchange. The evaluation was made for all nine possible combinations obtained by interchanging metals on the two surfaces. The results are graphically presented as a function of temperatures of the two surfaces. Equations of the form

$$q = a(T_1^b - T_2^b)\left(\frac{T_2}{T_1}\right)^c$$

were fitted to each of the nine sets of heat flux calculations, where q is the heat transfer rate and T_1 and T_2 are the temperatures of the hotter and cooler surfaces, respectively. Values of the constants, a, b, and c are presented along with contour plots showing the temperature regions in which the equations are accurate. A comparison with conventional calculation techniques is presented.

ANALYSIS

The equation and a computational procedure for evaluating nongray-body heat flux between two parallel, infinite surfaces of opaque material are described in references 1 and 2.

The basic method, frequently called the summation method, incorporates the following assumptions of conditions:

(1) Each surface is at a constant temperature.

(2) Each surface is a diffuse radiator.

(3) At a given temperature and wavelength the hemispherical spectral-absorptance equals the hemispherical spectral-emittance.

(4) A vacuum exists between the two plates.

The nongray-body net heat flux, q_{ng}, in watts per square centimeter is given by

$$q_{ng} = \int_o^\infty \frac{E_{b\lambda}(T_1) - E_{b\lambda}(T_2)}{\frac{1}{\epsilon_\lambda(T_1)} + \frac{1}{\epsilon_\lambda(T_2)} - 1} d\lambda \qquad (1)$$

where

$E_{b\lambda}(T)$ is the Planck distribution from a black body at temperature T.

$\epsilon_\lambda(T)$ is the spectral emittance of a surface at temperature T.

In the finite difference form required for machine calculation, the spectral emittance at wavelengths greater than 20 microns was assumed to be constant and the energy at wavelengths less than 0.2 microns was assumed negligible. With these assumptions, the finite difference form of Equation 1 is as given on the following page:

$$q_{ng} = \left[\sum_{\lambda=0.2}^{\lambda=20.0} \frac{\Delta\lambda}{(\epsilon_\lambda(T_1))^{-1} + (\epsilon_\lambda(T_2))^{-1} - 1} \left(E_{b\lambda}(T_1) - E_{b\lambda}(T_2)\right) \right.$$

$$+ \frac{1}{(\epsilon_{20}(T_1))^{-1} + (\epsilon_{20}(T_2))^{-1} - 1} \left[\left(\sigma T_1^4 - \sum_{\lambda=0.2}^{\lambda=20.0} E_{b\lambda}(T_1)\Delta\lambda\right)\right.$$

$$\left.\left. - \left(\sigma T_2^4 - \sum_{\lambda=0.2}^{\lambda=20.0} E_{b\lambda}(T_2)\Delta\lambda\right)\right]\right] \qquad (2)$$

The total hemispherical emittance was calculated in a similar manner; i.e., the same assumptions concerning the emittance at wavelengths larger than 20 microns and the energy below 0.2 microns were used. In both calculations, the size of the increment $\Delta\lambda$ was chosen constant at 0.05 microns.

The hemispherical spectral-emittance data to be used in the foregoing equations were computed from experimental measurements of normal spectral emittance [1]. These normal emittance values are presented in Table I. For tungsten and tantalum, metal temperatures were 1370°, 1920°, and 2480°K. Molybdenum data were obtained only at the two lower temperature values. The procedure of preparing these data for heat flux calculations was to convert the normal emittance values to hemispherical emittance using the method of Schmidt and Eckert [2]. These values were plotted for the range of 0.42 to 15 microns and extrapolated to include the range from 0.2 to 20 microns. Since the experimental data for molybdenum was available only at two temperatures, the values for other temperature were obtained by assuming the molybdenum spectral emittance had a non-linear variation with temperature similar to that of tungsten and tantalum. Hemispherical spectral-emittance of the three materials is presented in Figure 1. Calculated hemispherical total emittance results are shown in Figure 2.

TABLE I NORMAL SPECTRAL-EMITTANCE DATA OF REFERENCE I

Wavelength, λ, μ	Surface temperature, T, ^oK							
	1370	1920	2480	1370	1920	2480	1370	1920
	Normal spectral emittance, $\epsilon_{\lambda,T,n}$							
	Tungsten			Tantalum			Molybdenum	
.42	.478	.463	.456	.520	.511	.502	.406	.388
.54	.462	.446	.438	.494	.471	.459	.392	.375
.68	.445	.428	.416	.426	.412	.407	.374	.353
1.00	.377	.365	.356	.255	.291	.312	.301	.289
1.28	.314	.309	.309	.196	.238	.267	.233	.240
1.60	.249	.257	.266	.167	.205	.236	.179	.197
2.00	.181	.204	.224	.149	.183	.210	.141	.167
2.50	.135	.164	.191	.136	.164	.189	.117	.145
3.00	.111	.140	.169	.126	.153	.178	.103	.130
4.00	.0879	.116	.143	.114	.137	.158	.0895	.113
5.00	.0763	.102	.127	.106	.126	.143	.0793	.102
6.00	.0693	.0945	.116	.098	.117	.133	.0716	.0956
7.00	.0642	.0866	.109	.0919	.110	.127	.0676	.0892
8.00	.0606	.0821	.104	.0884	.104	.120	.0647	.0840
9.00	.0568	.0779	.101	.0825	.0989	.116	.0589	.0798
10.00	.0547	.0753	.0969	.0795	.0949	.114	.0577	.0766
11.00	.0526	.0717	.0945	.0745	.0918	.109	.0561	.0739
12.00	.0475	.0669	.0877	.0736	.0870	.105	.0526	.0696
13.00	.0454	.0677	.0890	.0680	.0876	.103	.0511	.0687
14.00	.0463	.0658	.0847	.0734	.0839	.102	.0529	.0690
15.00	.0458	.0653	.0847	.0535	.0824	.105	.0442	.0636

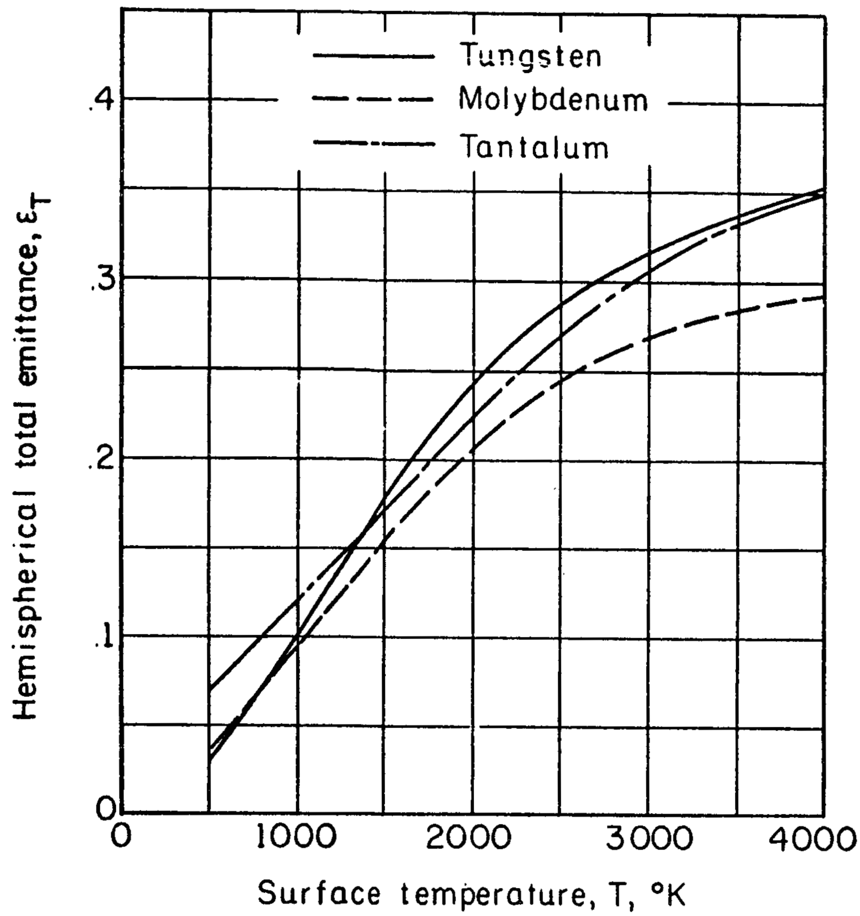


Figure 2. Hemispherical total-emittance data.

Using this data the nongray heat transfer data for the three metals were calculated as a function of the temperature difference between the two parallel plates. These results are presented in Figure 3.

Simple Equations for Net Heat Exchange

Simple expressions for net heat flux ease the mathematical complexity of analytical investigations. The conventional gray-body net heat transfer equation for parallel infinite constant temperature plates is;

$$q_g = \sigma \frac{T_1^4 - T_2^4}{\frac{1}{\epsilon_{T_1}} + \frac{1}{\epsilon_{T_2}} - 1} \quad (3)$$

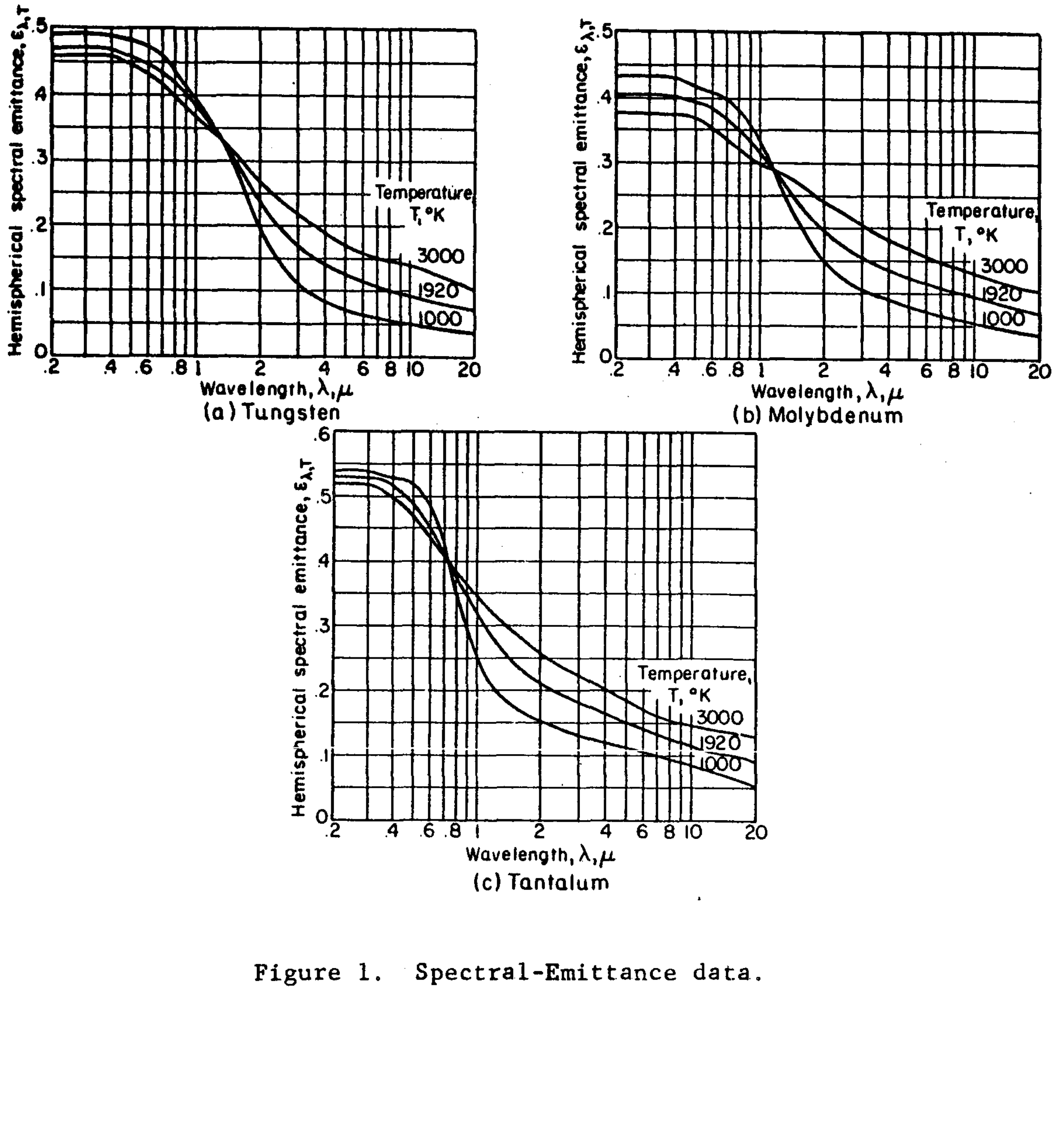
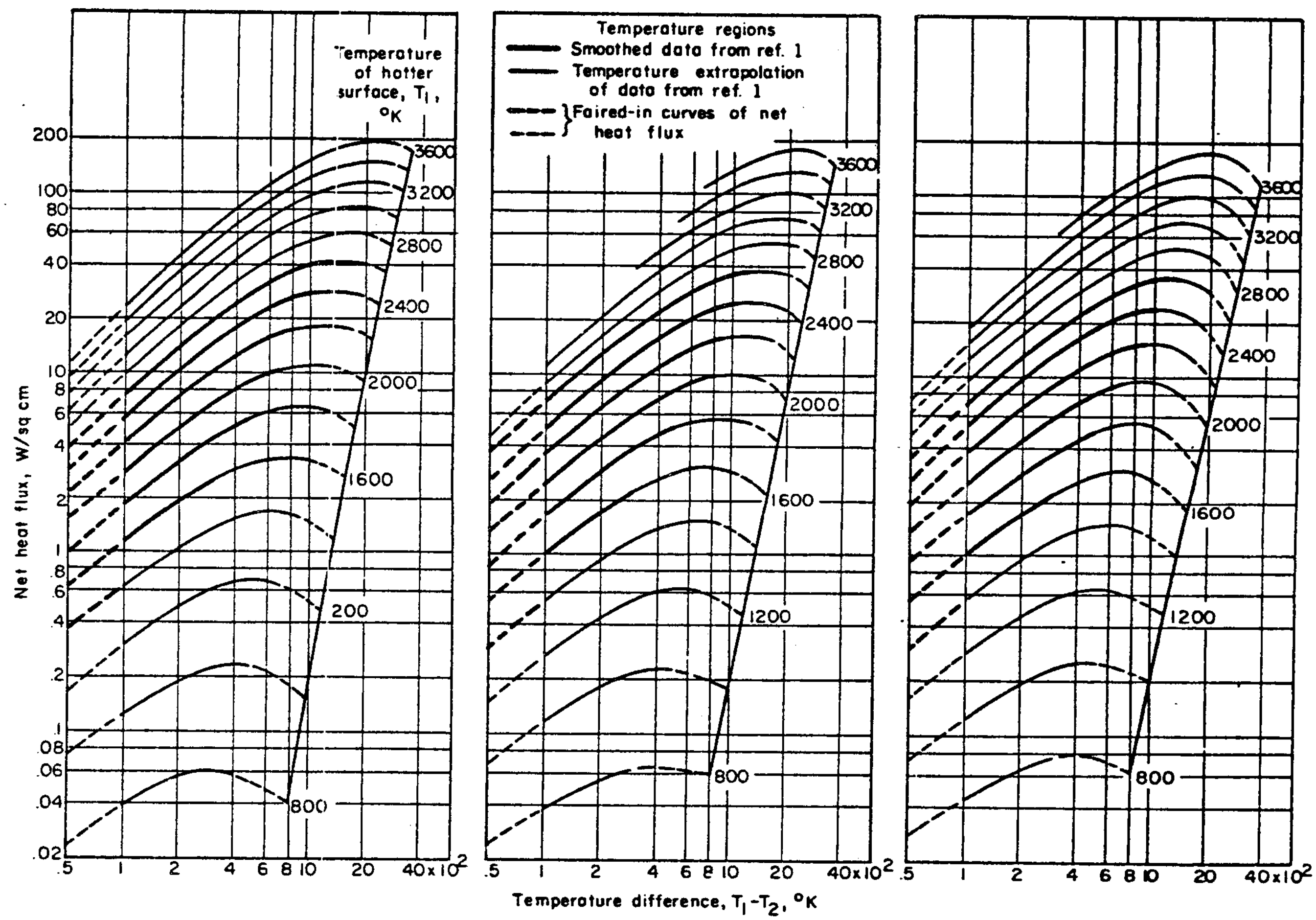
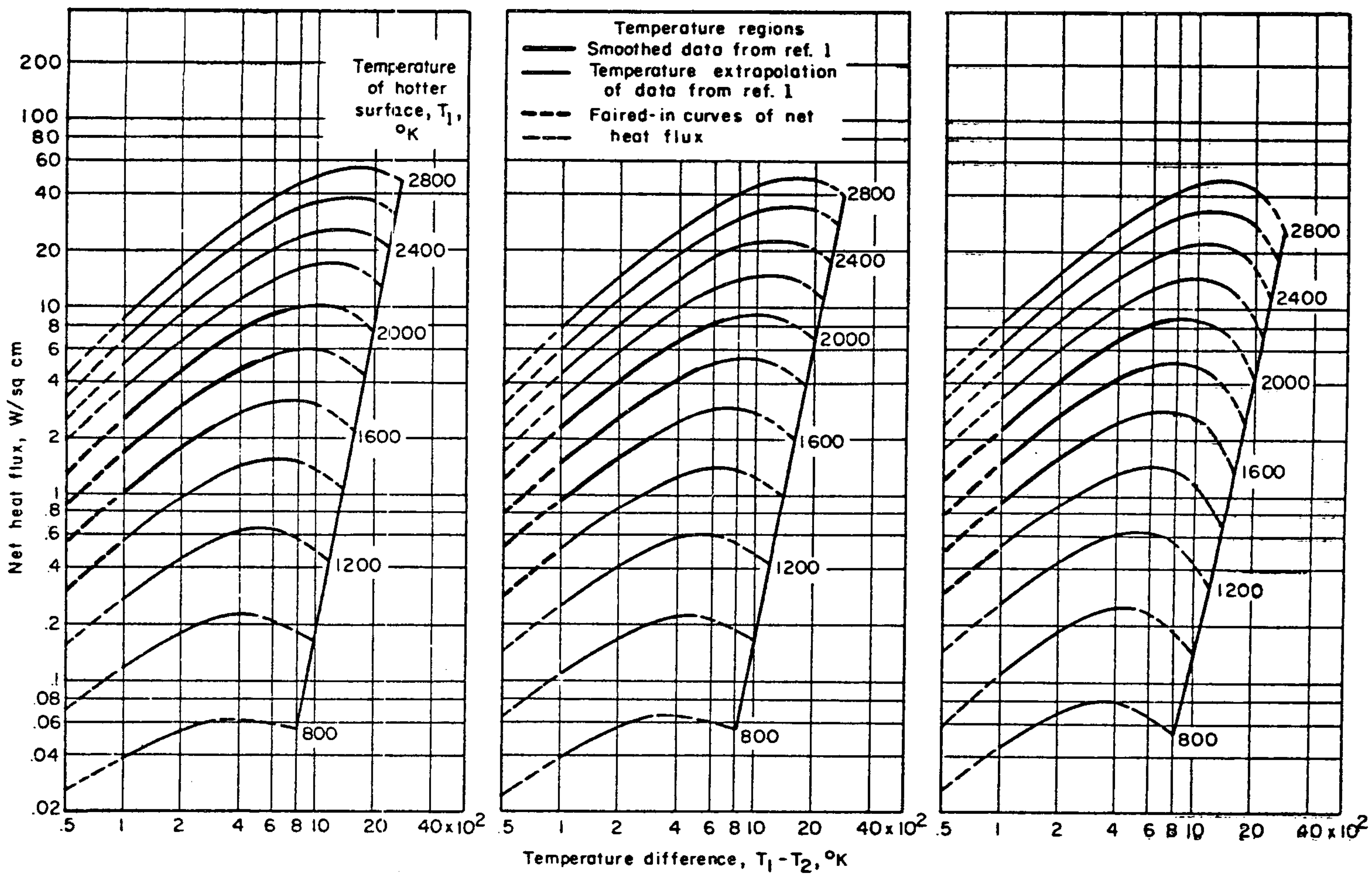


Figure 1. Spectral-Emittance data.



(a) Hotter surface, tungsten; colder surface, tungsten.
(b) Hotter surface, tungsten; colder surface, molybdenum.
(c) Hotter surface, tungsten; colder surface, tantalum.

Figure 3. Net radiant heat flux between two infinite, parallel plates

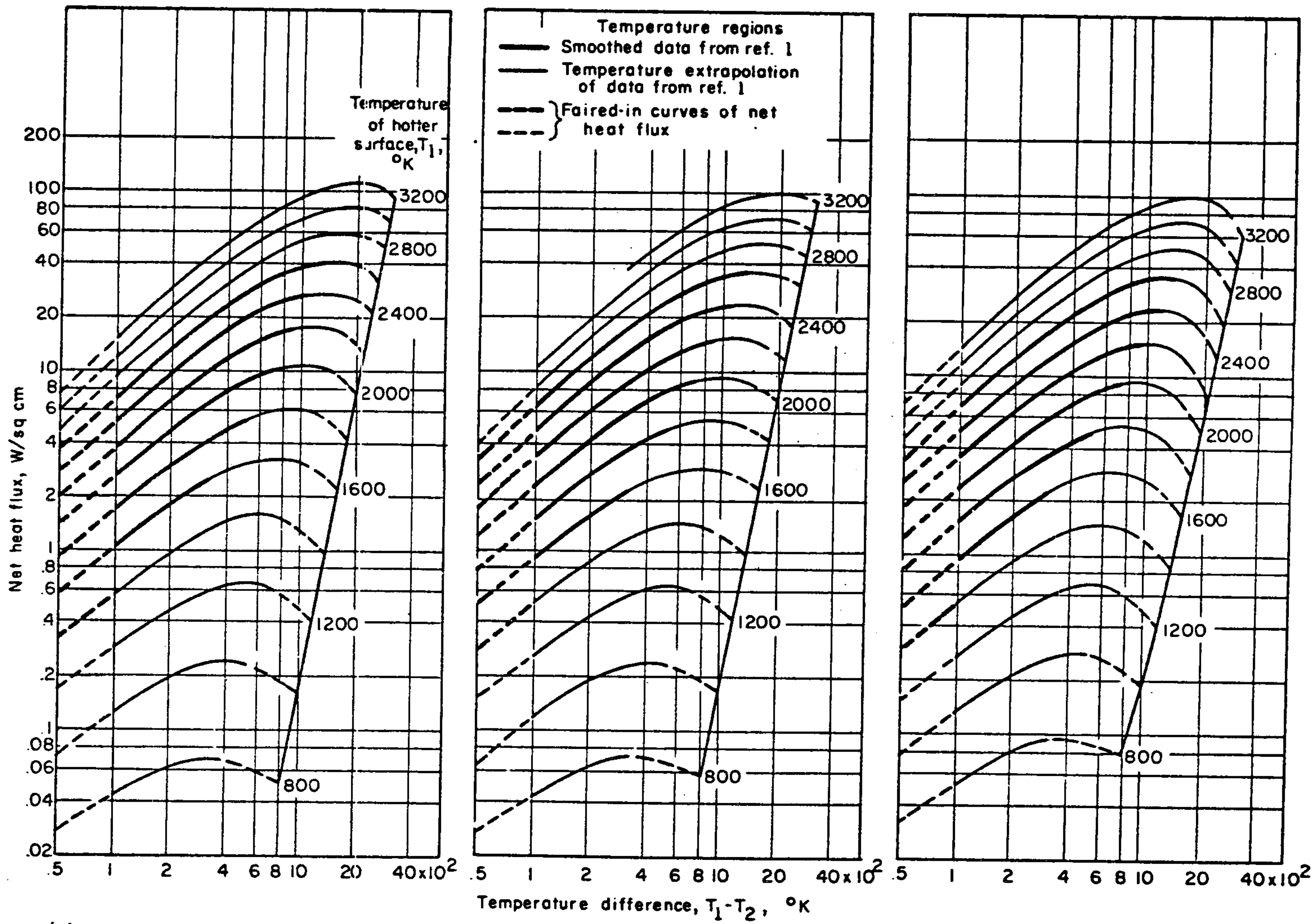


(d) Hotter surface, molybdenum; colder surface, tungsten.

(e) Hotter surface, molybdenum; colder surface molybdenum.

(f) Hotter surface, molybdenum; colder surface, tantalum.

Figure 3. Continued



(g) Hotter surface, tantalum; colder surface, tungsten.

(h) Hotter surface, tantalum; colder surface, molybdenum.

(j) Hotter surface, tantalum; colder surface, tantalum.

Figure 3. Concluded

Equation 3 is derived assuming the total emittance of a surface is equal to its absorptance. Since refractory metals do not adhere to this restriction, discrepancies between the results of this equation and Equation 1 are very large. The correlation can be improved by an empirical method [3] which repleaces ϵ_{T_2} in Equation 3 by an effective emittance of the colder surface ϵ_{T^*} evaluated at the geometric mean temperature,

$$T^* = \sqrt{T_1 \times T_2}$$

A comparison of the heat flux calculated using this method, q_g, to the heat flux calculated from Equation 1, q_{ng}, is shown in Figure 4a. These results are typical of the cases where tungsten or molybdenum was assigned to the colder surface. The nongray-body flux is larger than the gray-body flux and, in general, the error increases as T_1 is decreased and/or as ΔT is increased. When tantalum was the colder surface, typical results are shown in Figure 4b.

Expressions which would possibly provide better correlations than those determined by Equations 2 and 3 were examined. Of the several expressions investigated, the force fit Equation

$$q_{ff} = a(T_1^b - T_2^b) \left(\frac{T_2}{T_1} \right)^c \tag{4}$$

yielded flux values most closely matching the values of Equation 1. The constants a, b, and c for each set of metal surfaces were obtained as follows. The constant c was assigned a value of zero, then a best value for b was determined. This was accomplished by fixing b at each of a number of set values between 4 and 6, and then solving for a, while ranging variables T_1 and T_2. The best value of b was the one that produced a minimum deviation in the value of a. With b known, a best value for the constant c was determined in a manner similar to the determination of b. Values of c less than 0.1 had little influence and therfore were assigned a value of zero. Finally, with constants b and c established, the constant a was picked such that a close fit would be obtained over

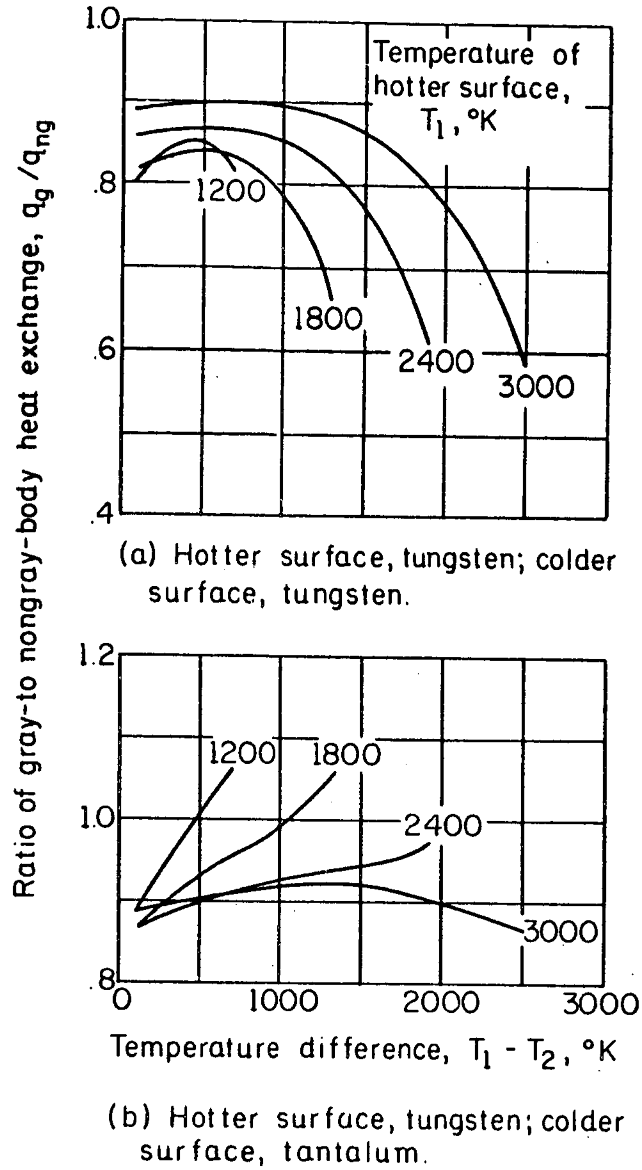


Figure 4. Comparison of gray and nongray body methods for computing net heat flux between two infinite parallel plates.

the widest useable range of temperatures. The results are presented in Table II and the contour plots illustrating equation accuracy are shown in Figure 5. When constants were being fitted to Equation 4, difficulties occurred in obtaining a reasonably satisfactory fit for several cases in which tungsten was the hotter surface. In each of these situations, two sets of constants are presented.

The values of flux given by the force fit equation were larger than the nongray-body values in the low temperature region. No

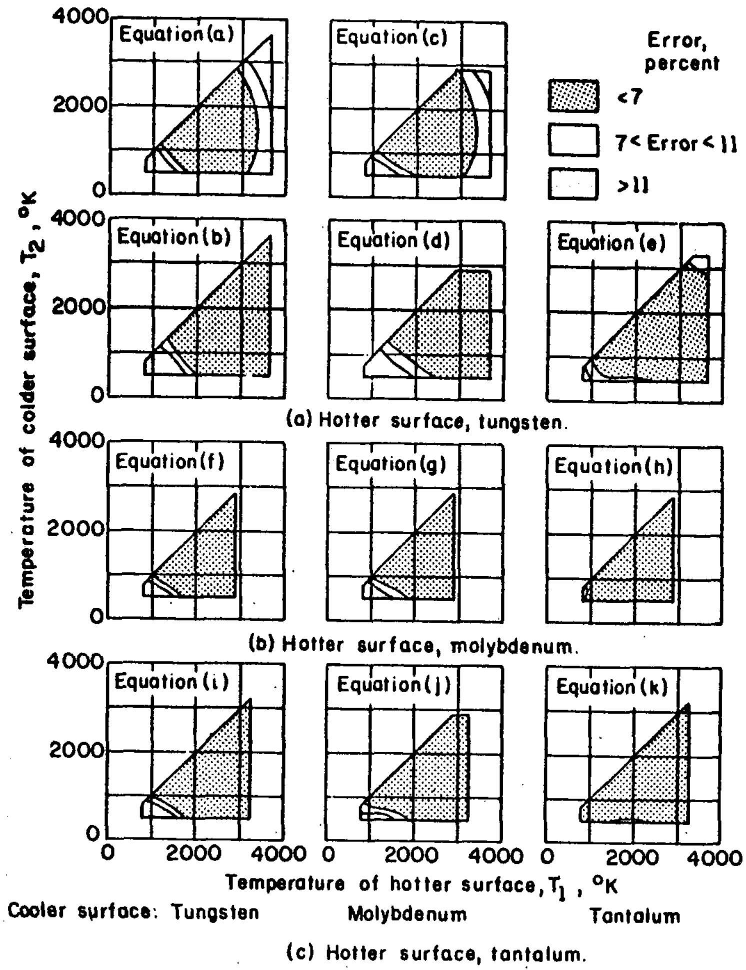


Figure 5. Error incurred with use of force-fit equations from table II.

attempt was made to improve the fit because of the lack of low temperature emittance data.

Comparison of Figures 4 and 5 shows that the force fit equations more accurately represent the results of nongray-body heat transfer calculations than the empirical gray-body equations. Also, in the shaded portions of Figure 5 (where the error is less than seven per cent) it is very likely that the heat transfer calculated from the force fit equation is as accurate as can be achieved in practice for real surfaces in a parallel plate geometry.

The results of Table II taken as a whole, show that the net heat exchange between two parallel, infinite surfaces of typical refractory metals is closely proportional to the fifth power of tem-

TABLE II. CONSTANTS OF FORCE-FIT EQUATION (EQ. (6))

$$q_{ff} = a\left(T_1^b - T_2^b\right)\left(\frac{T_2}{T_1}\right)^c$$

Hotter surface, T_1	Cooler surface, T_2	Constant			Temperature of hotter surface, T_1 °K	Equation (Fig. 5)
		a	b	c		
Tungsten	Tungsten	$.353\text{X}10^{-15}$	5.0	0	<3000	a
	Tungsten	$.166\text{X}10^{-14}$	4.8	0	>2000	b
	Molybdenum	$.314\text{X}10^{-15}$	5.0	0	<3000	c
	Molybdenum	$.149\text{X}10^{-14}$	4.8	0	>2000	d
	Tantalum	$.335\text{X}10^{-15}$	5.0	0.17	-----	e
Molybdenum	Tungsten	$.316\text{X}10^{-15}$	5.0	0	-----	f
	Molybdenum	.286	5.0	0	-----	g
	Tantalum	.308	5.0	0.17	-----	h
Tantalum	Tungsten	$.328\text{X}10^{-15}$	5.0	0	-----	i
	Molybdenum	.294	5.0	0	-----	j
	Tantalum	.326	5.0	0.17	-----	k

perature throughout the temperature range investigated. The temperature ratio factor is significant only for those cases in which tantalum is the cooler surface.

HOME PROBLEM STATEMENT

1. In Equation 2, all energy with wavelengths from 20 microns to longer wavelengths and from 0 to 0.2 are treated differently than the energy with wavelengths between 0.2 and 20 microns. Justify this division of the energy for 1000°K and 3000°K temperatures for the radiant energy sources.

2. Using Table I values, determine the total normal emittance of Molybdenum at 1920°K. Explain why this value does not agree with the value plotted in Figure 2.

3. Calculate the radiant exchange between a pair of infinite parallel tungsten plates at 2800°K and 1800°K using Equation 4 with the geometric mean temperature concept. Compare your answer to the nongray results.

REFERENCES

1. Branstetter, J. Robert: Radiant Heat Transfer Between Nongray Parallel Plates of Tungsten. NASA TN D-1088, 1961.

2. Branstetter, J.R.: Spectral Emissivities of Electrode Materials. Advanced Energy Conversion, Vol. 3, No. 1, Jan.-Mar., 1963.

3. Dreshfield, R.L.; and R.D. House: Investigation of the Spectral Normal Emittance of Single Crystals at High Temperatures. Rept. No. PWA-2306, Pratt & Whitney Aircraft, Feb. 15, 1964.

4. Jakob, Max: Heat Transfer. Vol. 1. John Wiley & Sons, Inc., 1949.

5. McAdams, W.H.: Heat Transmission. Third Edition, McGraw-Hill Book Co., Inc., 1954.

PROBLEM SOLUTION

1. The energy in the different wavelength regions can be compared easily by using the table of Planck's radiation functions.

For $T_1 = 1000^{o}K$ $\quad T_2 = 3000^{o}K$

$T_1 = 1800^{o}R$ $\quad T_2 = 5400^{o}R$

$\sigma T_1^4 = 1.97 \times 10^4 \frac{BTU}{hr\text{-}ft.^2}$ $\quad \sigma T_2^4 = .457 \times 10^6 \frac{BTU}{hr\text{-}ft.^2}$

From Planck's radiation functions,

λ (μ)	T (^{o}R)	λT (μ-^{o}R)	$\frac{E_{b(o-\lambda T)}}{\sigma T^4}$	$E_{b(0-.2)}$ (BTU/hr-ft^2)	$E_{b(2-20)}$ (BTU/hr-ft^2)	$E_{b(20-\infty)}$ BTU/hr-ft^2)
0.2	1800	360	0.0000	0 .0000		
0.2	5400	1080	0.0000	0 .0000		
20.0	1800	36,000	0.9840		1.94×10^4	
20.0	5400	108,000	0.9992		1.4599×10^6	
∞	1800	∞	1.000			315
∞	5400	∞	1.000			1100

Thus it is seen that the energy from both sources, in the range from 0 to 0.2, is essentially zero and can therefore be ignored. The energy from both sources, in the wavelength region beyond 20.0 microns, is seen to represent only a small fraction of the total energy emitted. Therefore, large variations in spectral emittance in this region would cause only very small changes in the total energy emitted. This is the justification for the assumption that the emittance is constant at wavelengths greater than 20.0 microns.

2. $$\epsilon_{T_N} = \frac{1}{\sigma T^4} \int_0^\infty \epsilon_{\lambda_N}(T) E_{b_\lambda}(T) d\lambda$$

The total normal emittance, ϵ_{T_N}, was determined by numerically evaluating the quantity

$$\frac{1}{\sigma T^4} \int_0^\infty \epsilon_{\lambda_N} E_{b_\lambda}(T) d\lambda$$

A computer program was utilized, and the values of $E_{b_\lambda}(T)$ were obtained from the table of Planck's radiation functions. The value of $\epsilon_{\lambda_N}(T)$ were taken from Table I.

The total normal emittance of molybdenum at 1920°K was found to be 0.17393. The values plotted in Figure 3 are total hemispherical emittance for the case considered is greater than the spectral normal emittance at every wavelength, it is to be expected that total hemispherical emittance be larger than total normal emittance.

3. $$q_{ff} = a\left(T_1^b - T_2^b\right)\left(\frac{T_2}{T_1}\right)^c$$

MTL. - Tungsten

$T_1 = 2800°$

$T_2 = 1800°$

Case I - $T_1 < 3000°K$

From Table III

$a = .353 \times 10^{-15}$

$b = 5$

$c = 0$

$q_{ff_I} = .353 \times 10^{-15} (2800^5 - 1800^5)$

$= .353 \times 10^{-15} [(2.8 \times 10^3)^5 - (1.8 \times 10^3)^5]$

$= .353(172 - 18.9)$

$= .353(153.1)$

$$\underline{\underline{q_{ff} = 54.1 \text{ W/cm}^2}}$$

Case II - $T_1 > 2000^\circ K$

From Table III

$a = .166 \times 10^{-14}$

$b = 4.8$

$c = 0$

$$q_{ff} = .166\times10^{-14}[2800^{4.8} - 1800^{4.8}]$$

$$= .166\times10^{-14}[(2.8\times10^{3})^{4.8} - (1.8\times10^{3})^{4.8}]$$

$$= .166\times10^{-14}(10^{14.4})(141 - 16.9)$$

$$= (.166)(10^{.4})(124.1)$$

$$\underline{\underline{q_{ff_{II}} = 59.6 \text{ W/cm}^2}}$$

From Figure 4(a), $T_1 - T_2 = 1000^\circ K$

$$\underline{\underline{q_{ng} = 54 \text{ W/cm}^2}}$$

$q_{ng}/q_{ff_I} = 54/54.1$

$$\underline{\underline{q_{ng}/q_{ff_I} = .997}}$$

```
C      PROGRAM TO EVALUATE TOTAL EMITTANCE AT TEMPERATURE T
C      FROM MONOCHROMATIC DATA
       DIMENSION  ALAMD(100), F1(100)
    2  FORMAT(1X,I3,F15.3,14H MATERIAL)
     6  FORMAT(8F10.5)
  38   FORMAT(30H ABOVE ARE N, T, AND MATERIAL )
  44   FORMAT(25H EMITTANCE OF MATERIAL AT, F9.2, 12H DEG. R. IS ,F8.5)
 100   READ(5,2) N,T
       READ(5,6) (ALAMD(I),E1(I),I=1,N)
       SUM1= 00.0
       DELA =100.0
    7  TLAM=1000.
    8  ALMD=TLAM/T
       L=N+1
    9  DO11I=1,N
       J=L-I
        IF(ALMD.GT.ALAMD(J)) GO TO 12
  11   CONTINUE
  12   IF(J.GE.N) GO TO 16
  13    I=J
 113   E1L=E1(I)+ (E1(I+1)-E1(I))*((ALMD-ALAMD(I))/(ALAMD(I+1)-ALAMD(I)))
        GO TO 21
  16   E1L=E1(N)
  21   TEM=(1.187E16)/((0.1714)*(TLAM**5.))
  22   FX=  25896./TLAM
  23   DNO =EXP(FX)-1.
  24   TEM= TEM/DNO
  25    SUM1=SUM1+E1L*DELA* TEM
       IF(TLAM.GE.14800.) GO TO 28
        TLAM=TLAM+200.0
       DELA =200.0
       GO TO 8
  28   IF(TLAM.GE.15000.) GO TO 30
       TLAM =TLAM + 200.00
       DELA = 600.0
       GO TO 8
  30   IF(TLAM.GE.29000.) GO TO 31
       TLAM = TLAM + 1000.0
       DELA = 1000.0
       GO TO 8
  31    IF(TLAM.GE.30000.) GO TO 32
       TLAM = TLAM + 1000.0
       DELA = 5500.0
       GO TO 8
  32   IF (TLAM.GE.90000.) GO TO 33
       TLAM =TLAM + 10000.0
       DELA = 10000.0
       GO TO 8
  33    IF (TLAM.GE. 100000.) GO TO 34
       TLAM =TLAM + 10000.0
       DELA = 5000.0
       GO TO 8
    34 WRITE(6,2) N,T
       WRITE(6,38)
       WRITE(6,44) T,SUM1
       GO TO 100
       END
```